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EVOLUTION OF A TURBULENT BURST

G. I. Barenblatt, N. L. Galerkina, and M. V. Luneva

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A complete solution of the problem of symmetric turbulent burst decay in a quiescent fluid is obtained using the semiempirical theory of turbulence.

The problem considered is of interest from many points of view. In homogeneous shear flows near a wall the intersection and self-intersection of eddies cause isolated, sharply defined turbulent bursts which completely determine the development of turbulence in the flow. This was convincingly demonstrated by the classical experiments of Kline's group at Stanford [1]. Moreover, turbulence in a stratified (with respect to density in a gravity field) ocean is patchy or "insular" [2]. The occurrence of patches of turbulence is associated with turbulent bursts resulting from the breaking of internal waves or local shear flow instability and subsequent mixing.

Generally speaking, the patches of turbulence associated with a burst are initially asymmetric. However, they rapidly acquire a symmetrical shape and, accordingly, the problem of the evolution of symmetrical turbulent patches is of fundamental importance. This problem is examined in the present article which, in addition to summarizing recent research, presents a number of new results. As always, most interest attaches to the intermediate-asymptotic stage of evolution of the burst, when the size of the turbulent patch is much greater than the initial value. In this stage the evolution of the patch obeys self-similar laws. Here, however, the self-similarity is nonclassical, so-called incomplete self-similarity (self-similarity of the second kind, scaling). Time enters into the self-similar variables to a power determined by the solution of the nonlinear eigenvalue problem. The solution of the problem is obtained within the framework of two variants of theory of turbulence of the Kolmogorov type [3, 4]: the classical (b, ℓ) variant and the (b, ε) variant [5-7]. It is an important advantage of the burst problem that, because of its symmetry, at the boundaries the turbulent energy fluxes, energy dissipation rates, etc., are equal to zero, so that there is no need to use additional (often very dubious) hypotheses to determine them. The results are similar, so that the solution obtained is also of interest from the standpoint of testing the Kolmogorov semiempirical theory for essentially unsteady flows.

1. Formulation of the Problem

It is proposed to consider (Fig. 1a) the evolution of a sudden burst of turbulence in a homogeneous quiescent fluid. The simplest symmetrical burst shape is a statistically uniform horizontal layer. Accordingly, we will investigate the decay in an unbounded quiescent

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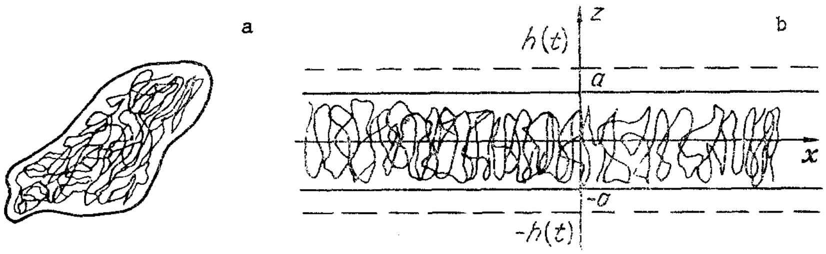

Fig. 1. Turbulent burst in a quiescent fluid: a) arbitrary shape; b) in the form of a statistically uniform horizontal layer.

fluid of a statistically uniform burst (Fig. 1b) initially enclosed between the horizontal planes z = a and z = −a. Let the initial turbulent energy per unit area of the layer boundary be

$$Q = \int_{-a}^{a} b(z,\ 0)\, dz. \tag{1}$$

Dimensionally, $[Q] = L^3T^{-2}$. Clearly, from the two kinematic quantities Q and a it is possible to construct a kinematic quantity of any dimensionality, so that, for instance, the initial conditions for the turbulent energy and the dissipation rate per unit mass can be written in the form:

$$b(z,\ 0) = (Q/a)\, u_0(s),\ \ \varepsilon(z,\ 0) = (Q^{3/2}/a^{5/2})\, v_0(s). \tag{2}$$

Here, s = z/a; $u_0(s)$ and $v_0(s)$ are dimensionless even functions identically equal to zero when $|s| \geq 1$.

Thus, at an arbitrary time t the kinematic characteristics of motion are determined by the following parameters:

$$Q,\ t,\ z,\ a, \tag{3}$$

the first two of which are dimensionally independent. By virtue of (3), for the turbulent energy b(z, t) and the rate of turbulent energy dissipation ε(z, t) per unit mass dimensional analysis gives the expressions

$$b = Q^{2/3}\, t^{-2/3} B(\xi,\ \eta),\ \varepsilon = Q^{2/3}\, t^{-5/3} E(\xi,\ \eta), \tag{4}$$

$$\xi = z/Q^{1/3}\, t^{2/3},\ \ \eta = a/Q^{1/3}\, t^{2/3}. \tag{5}$$

Here, B(ξ, η) and E(ξ, η) are dimensionless functions of their dimensionless arguments. From dimensional analysis, for the instantaneous half-thickness of the layer h(t) we obtain

$$h = Q^{1/3}\, t^{2/3} H(\eta), \tag{6}$$

since, obviously, h does not depend on z; H(η) is a dimensionless function of its dimensionless argument. Of particular interest is the solution for large times, when the thickness of the layer is much greater than its initial value: h(t) ≫ a, i.e., the asymptotic form of the solution (4), (6) when η ≪ 1.

2. Asymptotic Form of Solution

The assumption first in degree of complexity is that when η ≪ 1 we have complete self-similarity with respect to the parameter η (for more details concerning the concepts of complete and incomplete self-similarity see [8]). This assumption implies that as η → 0 there exist finite nonzero limits of the functions B(ξ, η), E(ξ, η) and H(η):

$$\lim B(\xi,\ \eta) = B(\xi),\ \lim E(\xi,\ \eta) = E(\xi),\ \lim H(\eta) = \xi_0, \tag{7}$$

so that the solution (4), (6) takes the form:

$$b = Q^{2/3}\, t^{-2/3} B(\xi),\ \ \varepsilon = Q^{2/3}\, t^{-5/3} E(\xi),\ h = \xi_0 Q^{1/3}\, t^{2/3}. \tag{8}$$

However, this assumption is incorrect. Thus, let us take the turbulent energy balance equation which in the case of a statistically uniform flow without shear has the form:

$$\partial_t b + \partial_z q_b = -\varepsilon, \tag{9}$$

and integrate it with respect to z from $z = -h(t)$ to $z = h(t)$, taking into account the fact that at the boundaries of the turbulent layer $z = \pm h(t)$ both the turbulent energy and its flux vanish:

$$b(h(t), t) = 0, \; q_b(h(t), t) = 0. \tag{10}$$

We find that the rate of change of energy per unit area of the layer is negative:

$$\frac{d}{dt}\int_{-h}^{h} b(z, t)\,dz = -\int_{-h}^{h} \varepsilon(z, t)\,dz < 0, \tag{11}$$

since in the region of the turbulent layer the dissipation rate ε is positive. At the same time, the solution (8) shows that the energy of unit area of the layer

$$\int_{-h}^{h} b(z, t)\,dz = \frac{Q^{2/3}}{t^{2/3}}\int_{-h}^{h} B(\xi)\,dz = Q\int_{-\xi_0}^{\xi_0} B(\xi)\,d\xi = \text{const}\,Q \tag{12}$$

does not depend on time. This contradiction demonstrates the incorrectness of the assumption of complete self-similarity.

We will therefore assume incomplete self-similarity (self-similarity of the second kind, scaling) with respect to the parameter η, i.e., the existence of power-law asymptotic forms of the functions $B(\xi, \eta)$, $E(\xi, \eta)$ and $H(\eta)$ as $\eta \to 0$ in the absence of a finite nonzero limit of these functions:

$$B = \eta^{\lambda_1} F(\xi/\eta^{\nu_1}),\; E = \eta^{\lambda_2} G(\xi/\eta^{\nu_1}),\; H = \text{const}\,\eta^{\nu_2}. \tag{13}$$

The dimensionless constants λ_1, λ_2, ν_1, and ν_2 are related by expressions that follow from the fact that the functions F and G depend on the same variable ξ/η^{ν_1}, and the quantities b, q_b, and ε satisfy the turbulent energy balance equation (9). Using these relations and Eqs. (4) and (13), we find that the distributions of the turbulent energy and turbulent energy dissipation rate and the thickness of the half-layer may be expressed as follows:

$$b = A^2 t^{-2\mu} f(\zeta),\; \varepsilon = A^2 t^{-2\mu-1} g(\zeta),\; h = At^{1-\mu}. \tag{14}$$

Here, f = const F, g = const G,

$$\zeta = z/h = z/At^{1-\mu},\; \mu = (1 + 2\nu_1)/3,\; A = \text{const}\,Q^{(1-\nu_1)/3} a^{\nu_1}. \tag{15}$$

The parameter μ, which determines the law of expansion of the layer and the rate of decay of the turbulence, cannot be established from dimensional considerations. In order to determine it, it is necessary to formulate and solve the eigenvalue problem.

3. Eigenvalue Problem: (b, ε) Model

The (b, ε) model is based on the use of the turbulent energy dissipation rate balance equation in addition to the turbulent energy balance equation. In the case of a statistically uniform horizontal turbulent layer for zero mean flow velocity this equation takes the form [7]:

$$\partial_t \varepsilon + \partial_z q_\varepsilon = -U. \tag{16}$$

The dissipation rate flux q_ε and the "rate of equalization of the dissipation rate" U are one-point moments of the velocity fluctuation field and the gradient of that field.

We introduce the turbulent transfer coefficients for turbulent energy k_b and dissipation rate k_ε:

$$k_b = -q_b/\partial_z b,\; k_\varepsilon = -q_\varepsilon/\partial_z \varepsilon. \tag{17}$$

We note that in the case of a statistically uniform layer relations (17) do not contain any additional hypothesis. According to the Kolmogorov hypothesis, in developed turbulent flow the turbulent eddy field is self-similar. Therefore, correct to the constant multipliers, all the kinematic characteristics of that field are determined by two of these quantities having different dimensions. Taking as these determining characteristics b, $[b] = L^2T^{-2}$; ε, $[\varepsilon] = L^2T^{-3}$, we obtain a closed system of equations, the (b, ε) model. Thus, in this case dimensional analysis gives

$$k_b = \alpha b^2/\varepsilon,\; k_\varepsilon = \beta b^2/\varepsilon,\; U = \gamma \varepsilon^2/b, \tag{18}$$

so that equations (9) and (16) take the form:

$$\partial_t b = \alpha \partial_z [(b^2/\varepsilon)\,\partial_z b] - \varepsilon, \tag{19}$$

$$\partial_t \varepsilon = \beta \partial_z [(b^2/\varepsilon)\, \partial_z \varepsilon] - \gamma \varepsilon^2/b. \tag{20}$$

The coefficients α, β, and γ are found by comparison with experiment in certain standard problems. Some interesting solutions of unsteady problems using the (b, ε) model were given in [9-11]. We note that the problems considered in those studies were nonsymmetric.

Substituting the solution in form (14)-(15) in Eqs. (19) and (20) and, for convenience, taking as the unknowns $f = \beta F$ and $g = \beta G$, we obtain for the functions f and g the system of ordinary differential equations

$$\frac{\alpha}{\beta}\,\frac{d}{d\zeta}\left[\frac{f^2}{g}\,\frac{df}{d\zeta}\right] + (1-\mu)\,\zeta\,\frac{df}{d\zeta} + 2\mu f - g = 0, \tag{21}$$

$$\frac{d}{d\zeta}\left[\frac{f^2}{g}\,\frac{dg}{d\zeta}\right] + (1-\mu)\,\zeta\,\frac{dg}{d\zeta} - \gamma\,\frac{g^2}{f} + (1+2\mu)\,g = 0. \tag{22}$$

In view of the symmetry of the layer with respect to the z coordinate, it is possible to consider only half the layer $(0 \leq z \leq R(t))$ and impose the boundary conditions on the boundaries $z = 0$ and $z = h(t)$. Clearly, on the boundary $z = h(t)$ the turbulent energy b, the dissipation rate ε and their fluxes q_b and q_ε must be continuous. Outside the turbulent layer the fluid is quiescent. Therefore, at $z = h(t)$ the following conditions are satisfied:

$$b = 0,\ \varepsilon = 0,\ q_b = 0,\ q_\varepsilon = 0, \tag{23}$$

whence and from (17)-(18) we find

$$b = 0,\ \varepsilon = 0,\ (b^2/\varepsilon)\,\partial_z b = 0,\ (b^2/\varepsilon)\,\partial_z \varepsilon = 0 \text{ when } z = h(t). \tag{24}$$

Using the representation of the solution (14), we obtain the first group of boundary conditions for the system (21)-(22):

$$f = g = 0,\ (f^2/g)\,df/d\zeta = 0,\ (f^2/g)\,dg/d\zeta = 0 \text{ when } \zeta = 1. \tag{25}$$

Then, by symmetry, in the middle of the layer, i.e., at $z = 0$, the fluxes are equal to zero:

$$q_b = q_\varepsilon = 0 \text{ when } z = 0, \tag{26}$$

whence and from (17), (18) we obtain the second group of boundary conditions for the system (21)-(22):

$$(f^2/g)\,df/d\zeta = 0,\ (f^2/g)\,dg/d\zeta = 0 \text{ when } \zeta = 0. \tag{27}$$

We note that for nonsymmetric problems (see [9-11]) the formulation of conditions analogous to (27) requires the introduction of important additional hypotheses.

Thus, we have obtained a boundary value problem (25), (27) for the system of second-order ordinary differential equations (21), (22) containing only one parameter μ that is not known in advance. Simple estimates show that the values of the parameter μ lie on the interval $1/3 \leq \mu \leq 1$. Moreover, the values of the parameters α and β recommended in [7, 10, 11] give a value of the ratio α/β that falls generally between 0.7 and 1.2. Below, we will confine ourselves to the simplest case $\alpha = \beta$; the complications which arise when $\alpha/\beta \neq 1$ are not of fundamental importance.

The boundary-value problem (21), (22), (25), (27) possesses the following property: it is invariant under the one-parameter group of continuous transformations:

$$f_1(\zeta_1) = \omega^{-2} f(\zeta),\ g_1(\zeta_1) = \omega^{-2} g(\zeta),\ \zeta_1 = \omega^{-1}\zeta \tag{28}$$

(ω is the group parameter). This property enables us to reduce the order of Eqs. (21), (22) and carry out a qualitative investigation of their solutions. The investigation shows that there exists a class of solutions f, g positive when $\zeta < 1$, identically equal to zero when $\zeta \geq 1$, continuous and having continuous values of $(f^2/g)df/d\zeta$ and $(f^2/g)dg/d\zeta$. In the neighborhood of the point $\zeta = 1$ when $\zeta < 1$ the solution can be expanded as follows:

$$f = c(1-\mu)(1-\zeta) + \ldots,\ g = c^2(1-\mu)(1-\zeta) + \ldots \tag{29}$$

Here, the positive quantity c is, together with μ, a parameter of the problem. Each pair c, μ uniquely determines a nontrivial solution of the system (21), (22) satisfying conditions (25). It is necessary to find the values of the parameters c, μ for which the solution also satisfies conditions (27). Thus, as is usually the case with self-similar solutions

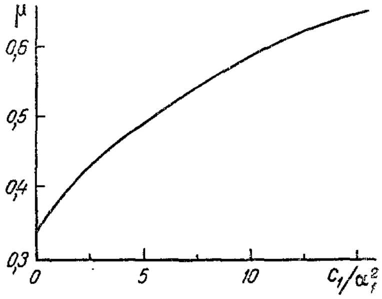

Fig. 2. Eigenvalue of the problem for the (b, l) model as a function of the parameter c_1/α_1^2.

of the second kind [8], we have arrived at a nonlinear eigenvalue problem for determining the exponents of time in the self-similar variables.

This problem can easily be solved numerically and corresponding calculations have been carried out. They confirmed that for each value of the parameter $\gamma > 1.5$ there exists a unique solution of the eigenvalue problem formulated. The calculations also revealed an important property of the solution constructed: over the entire interval from $\zeta = 0$ to $\zeta = 1$ the ratio g/f is constant and equal to c, which makes it possible to obtain a solution in simple finite form. We set g = cf and substitute this relation in Eqs. (21), (22). We obtain two equations for the single function f, which coincide only if $c = (\gamma - 1)^{-1}$. In this case the equation for f takes the form:

$$\frac{1}{c}\frac{d}{d\zeta}\left[f\frac{df}{d\zeta}\right]+(1-\mu)\zeta\frac{df}{d\zeta}+2\mu f-cf=0. \tag{30}$$

Integrating both sides from $\zeta = 0$ to $\zeta = 1$ and using boundary conditions (25), (27), we find

$$(1-3\mu+c)\int_0^1 f d\zeta=0, \text{ i.e. } \mu=\frac{1+c}{3}=\frac{\gamma}{3(\gamma-1)}, \tag{31}$$

since the function f is positive and its integral is positive. Substituting the values $c = 1/(\gamma - 1)$ and $\mu = \gamma/3(\gamma - 1)$ in (30), we obtain an equation which is easy to integrate with conditions (25), (27); the solution is found in finite form:

$$f=D(1-\zeta^2),\ g=D(1-\zeta^2)(\gamma-1)^{-1},\ 0\leqslant\zeta\leqslant1, \\ f=g\equiv0,\ \zeta>1,\ D=[(2/3)\gamma-1]/2(\gamma-1)^2. \tag{32}$$

Numerical calculation of the general non-self-similar problem for Eqs. (19), (20) with conditions (2), (24), (26) and various functions $u_0(s)$, $v_0(s)$ in the initial condition (2) showed that the solution of the non-self-similar problem goes over into the self-similar asymptotic regime (14), (31), (32). Thus, for example, if we take the value of the parameter $\gamma = 2$ recommended in [7], the asymptotic solution may be represented in the form:

$$b=(1/6\alpha)Q^{1/3}at^{-4/3}(1-\zeta^2),\ \varepsilon=(1/6\alpha)Q^{1/3}at^{-1/3}(1-\zeta^2), \\ \zeta=z/h,\ h=Q^{1/6}a^{1/2}t^{1/3}. \tag{33}$$

Here, we have made use of the fact that for the numerical calculations in question the value of const in expression (14) for the constant A is very close to unity. We recall that for self-similar solutions of the second kind the constant parameters of this type are determined (see [8]) not from the conservation laws but from the matching of the self-similar asymptotic form and the numerical solution of the non-self-similar problem. The numerical calculations performed also showed that for values of α/β on the interval of interest the value of the parameter μ differs only slightly from 2/3.

4. Eigenvalue Problem: (b, ℓ) Model

The (b, ℓ) model is based on the turbulent energy balance equation (9) and the same Kolmogorov self-similarity hypothesis. However, the turbulent energy b and the scale of turbulence ℓ are taken as the determining kinematic quantities. In accordance with the developed turbulence self-similarity hypothesis we obtain

$$k_b=l\sqrt{b},\ \varepsilon=c_1b^{3/2}/l, \tag{34}$$

where c_1 is a constant; the constant in the expression for k_b can be taken equal to unity by renormalizing the scale of turbulence. In this case we use only the turbulent energy balance equation, which by virtue of (34) reduces to the form:

$$\partial_t b = \partial_z l \sqrt{b}\, \partial_z b - c_1 b^{3/2}/l. \qquad (35)$$

In order to close it, in the problem under consideration we make the simple assumption $\ell = \alpha_1 h$, α_1 = const: the scale is constant over the thickness of the layer and proportional to the thickness of the layer. By making this assumption we neglect the time required for the adaptation of the eddies to the dimensions of the layer.

As before, the asymptotic solution is represented in the form (14); for determining the function f and the parameter μ from Eq. (35) and conditions (10) and, moreover, the symmetry condition we obtain the eigenvalue problem

$$\frac{d}{d\zeta}\left[\alpha_1 \sqrt{f}\, \frac{df}{d\zeta}\right] + (1-\mu)\,\zeta\, \frac{df}{d\zeta} + 2\mu f - c_1 \frac{f^{3/2}}{\alpha_1} = 0, \qquad (36)$$

$$\frac{df^{3/2}(0)}{d\zeta} = 0,\ f(1) = 0,\ \frac{df^{3/2}(1)}{d\zeta} = 0. \qquad (37)$$

Because of the property of invariance under a certain group of transformations the eigenvalue μ proves to depend only on the combination c_1/α_1^2. The problem can easily be solved numerically; a graph of the function $\mu(c_1/\alpha_1^2)$ is reproduced in Fig. 2.

Let us compare the solutions corresponding to the (b, ε) and (b, ℓ) models and estimate the constants α_1 and c_1. Numerical calculations show that the value μ = 2/3 obtained for the (b, ε) model corresponds to $c_1/\alpha_1^2 \simeq 17$. According to (34), for the (b, ℓ) model we have: $\ell = c_1 b^{3/2}/\varepsilon$, so that $k_b = \ell\sqrt{b} = c_1 b^2/\varepsilon$. On comparing this with (18) we obtain $c_1 = \alpha$. Hence it follows that $\alpha_1 = (\alpha/17)^{1/2}$. Introducing the value of α recommended in [7], we obtain c_1 = 0.064-0.085, α_1 = 0.063-0.071. We note that the parameter c_1 is close to the value c_1 = 0.062 obtained for a layer of constant shear stress [4].

Thus, we have presented solutions of the problem of the evolution of a turbulent burst in the form of a plane layer obtained within the framework of the principal models of the semiempirical theory. Solutions for cylindrical (μ = 0.75) and spherical (μ = 0.8) bursts can be obtained in the same way. The solutions obtained using the two models are similar in form. The most important difference with respect to the solution obtained within the framework of the (b, ε) model is the vanishing of the scale of turbulence $\ell = c_1 b^{3/2}/\varepsilon$ near the boundary of the layer (cf. (34)) and the sharper fall in turbulent energy near the boundary.

NOTATION

a, initial half-thickness of the burst; b(z, t), turbulent energy per unit fluid mass; h(t), variable half-thickness of the burst; k_b, turbulent transfer coefficient for turbulent energy; k_ε, turbulent transfer coefficient for dissipation rate; ℓ, scale of turbulence; q_b, turbulent energy flux; q_ε, dissipation rate flux; t, time; x, longitudinal coordinate of the burst; z, transverse coordinate of the burst; Q, initial turbulent energy per unit area of the burst boundary; U, rate of equalization of dissipation rate; ε, rate of dissipation of the turbulent energy of unit mass; s, ξ, η, ζ, ζ_1, dimensionless variables defined by position; $u_0(s)$, $v_0(s)$, B(ξ, η), B(ξ), E(ξ, η), E(ξ), H, F, G, f, g, f_1, g_1, dimensionless functions defined by position; A, c, c_1, α, α_1, β, γ, λ_1, λ_2, ν_1, ν_2, ξ_0, ω, constant parameters defined by position.

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FLOW AND HEAT TRANSFER OF FINELY DISPERSED TURBULENT FLOWS IN CHANNELS

I. V. Derevich, V. M. Eroshenko, and L. I. Zaichik

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Equations for the second moments of the velocity and temperature fluctuations are used to study the effect of particles on the rate of turbulent momentum and heat transfer in the flow of a gas suspension in circular pipes.

It is currently most promising to describe the hydrodynamics and heat transfer of turbulent disperse flows by using the system of equations for the second one-point moments of the velocity and temperature fluctuations of the dispersion medium with allowance for the presence of the particles [1-4]. The authors of [5-8] used this system to analyze the effect of the disperse phase on the fluctuation and mean flow and heat-transfer characteristics of dust-laden flows in channels for particles for which the dynamic and thermal relaxation times were of the same order as the integral turbulence scale. The present investigation, being a continuation of [5-8], studies the manner in which the rate of turbulent transport is affected by finer particles, having a dynamic relaxation time which is one order less than the microscopic time scale of the turbulence. We will also present results of calculations of the hydrodynamics and heat transfer of dust-laden flows within a broad range of particle dimensions.

1. We are examining the turbulent flow of a gas with spherical solid particles ($\rho_2 \gg \rho_1$). The system of equations of motion and heat transfer of the gas in the case of a small volume content of particles has the form:

$$\frac{\partial U_i}{\partial t} + U_k \frac{\partial U_i}{\partial x_k} = -\frac{1}{\rho_1}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_k \partial x_k} - \frac{\rho_2}{\rho_1}\frac{\omega}{\Omega_N}\sum_{p=1}^{N} \delta(\mathbf{x} - \mathbf{R}_p(t))\frac{dV_{pi}(t)}{dt}, \tag{1}$$

$$\frac{\partial \Theta_1}{\partial t} + U_k \frac{\partial \Theta_1}{\partial x_k} = \chi \frac{\partial^2 \Theta_1}{\partial x_k \partial x_k} - \frac{\rho_2 c_2}{\rho_1 c_1}\frac{\omega}{\Omega_N}\sum_{p=1}^{N} \delta(\mathbf{x} - \mathbf{R}_p(t))\frac{d\Theta_p(t)}{dt}, \tag{2}$$

$$\frac{dV_{pi}}{dt} = \frac{1}{\tau_u}(U_i(\mathbf{R}_p(t),\ t) - V_{pi}(t)),\quad \frac{dR_{pi}}{dt} = V_{pi}, \tag{3}$$

$$\frac{d\Theta_p}{dt} = \frac{1}{\tau_\theta}(\Theta_1(\mathbf{R}_p(t),\ t) - \Theta_p(t)). \tag{4}$$

If we change over from a Lagrangian description of the individual particles (3), (4) to an Eulerian description of the solid phase [8], average Eqs. (1) and (2) and the equations obtrained for the solid phase in the case of turbulent flow, and add up the equations for the individual phases, we obtain the equations of motion and heat transfer for the disperse flow as a whole:

G. M. Krzhizhanovskii Scientific-Research Institute of Power Engineering, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 5, pp. 740-751, November, 1987. Original article submitted March 5, 1987.

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